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Comparative study of FEM and XFEM

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ABSTRACT: Classical Finite Element Method is a numerical approach to solve the engineering analysis but FEM in not well for the problem that having any type of discontinuity in it may be of strong or weak in nature because the FEM must have the conformal mesh with the discontinuity so the Extended Finite Element Method has been developed to minimize the requirement of conformal mesh and XFEM is able to handle accurately the strong and weak discontinuity without conformal mesh. In this work XFEM is proposed and applied to analyse the 2D problem. 2D problem of a plate with hole has been analysed by XFEM. Various sizes of hole in plate have been taken and analysed for the evaluation of stress concentration factor by XFEM and various models of plate with hole has been analysed by XFEM.

***Keywords**:* Plate with hole, Extended Finite Element Method, finite element analysis.

I. INTRODUCTION

The design of mechanical component is a step to step methodology starting from the recognition of need to production. Analytical design of a mechanical component is difficult and time consuming. During design of any mechanical component it should kept in mind that the component should be deformed only under the permissible stress when we applied the force on it, so during design we have to study the behaviour of component under the load. So the mathematical analysis of the component will be carried out. The mathematical analysis is expressed in terms of the partial differential equations, hence to know the behaviour we have to solve these partial differential equations. For solving these equations we will use numerical analysis. Numerical analysis is the study of algorithms that use numerical approximation for the problems of mathematical analysis. The numerical analysis includes all the method that can reproduce the system in an analogy or digital fashion.

A. Background

There are various numerical analysis methods like Finite Element Method, Finite Difference Method, Boundary Element Method, Generalized Finite Element Method Mesh free method. These are the some numerical analysis methods which are solving the partial differential equation to explain the behaviour of the mechanical component under applied force. Basis on the result of the method which we are used the design process of the mechanical component will be carried out.

Every method in the above list has unique characteristics to apply on the component to find the behaviour of the component. To overcome these difficulties related to FEM, a novel approach known as extended Finite Element Method (XFEM) has been recrudesced to model arbitrary discontinuities without conformal mesh or re-meshing.

FEM has shown to be of insufficient applicability in solving problems with types such as interior boundaries, discontinuities or singularities because of the need of re-meshing and high mesh densities in it. FEM is requires that the mesh creation from the geometry so FEM requires that the hole surface coincides with the edge of the finite elements i.e. a conformal meshing will be essential require. To overcome these difficulties, a well conformed numerical technique known as extended Finite Element Method (XFEM) has been developed to model arbitrary discontinuities without conformal mesh or re-meshing. The extended finite element method (XFEM) is a numerical method for the approximation of solutions that involve jumps, kinks, singularities, or general high gradients in some parts of the domain. Extended Finite Element Method is allowing the modeling of arbitrary geometric features independently of the finite element mesh. In XFEM the discontinuities like crack or hole is easily identified or handle by Level Set function, which allow the discontinuities to be defined in depended of the mesh.

II. PROBLEM DEFINITION

A rectangular domain with circular hole under mechanical load has been considered. The numerical simulation has been accomplished for calculation of Stress Concentration factor by using of following data

- Modulus of Elasticity (E) = 200×10^3 N/mm^2
- Poisson Ratio (ʋ) = 0.3
- Far Field Stress (σ_0) = 100 N/mm^2

The results have been calculated by using of four nodes Quadrilateral Element.

A hole is present in the plate as a discontinuity in it. Stress Analysis is carried out of this model firstly by using classical Finite Element Method (FEM) and after that the solution obtained from FEM for comparing the Calculated Stress Concentration Factor with the Peterson charts for validation process. The same model has been studied by Appling "Extended Finite Element Method", XFEM.

A. Standard FEM Solution using Conformal Mesh

The Problem has been solved using standard Finite Element Method. In this case A conformal mesh is required at every stage of the analysis and high accuracy is required during mesh generation process. The problem is analyzed by using FEM with four nodes Quadrilateral Element. The dimension of the Plate are as given below

Length=200 mm, Width=100 mm, Thickness=1 mm, Pressure applied=100 N/mm2, E=200×10^3 N/mm^2
Possion ratio = 0.3

The hole in the plate is considered varying in size and we will consider the d/w ratio means Diameter of hole to the width of the plate to calculation of SCF around the hole and variation of SCF accordingly to the hole size or d/w ratio are also being studied. The Standard FEM analysis of the problem is being carried out by using ANSYS software.

III. RESULT AND DISCUSSION

Case I when d/w ratio is 0.2 means Diameter of hole present in the plate is 20 mm at the centre of the plate the FEA analysis is being carried out and maximum value of stress is find out from the solution obtained from ANSYS software after that for finding Stress Concentration factor we have to find out the nominal stress because we know that the Stress Concentration Factor (K_t) is equal to Maximum Stress divided by Nominal Stress in the problem hence $K_t = \frac{\sigma_{Maximum}}{\sigma_{Nominal}}$

where $\sigma_{Maximum}$ is the maximum stress occur at the edge of the hole and $\sigma_{Nominal}$ is the nominal stress. Nominal stress is just the gross stress in the same element under the same loading conditions without the stress concentrators (holes, notches, shoulders and so on). From above diagram it is clear that the maximum stress is obtained at the edge of the hole. The maximum value of the stress is 315.6 N/mm^2 and nominal stress is equal to 125 N/mm^2. So the stress concentration at the hole is 2.52.

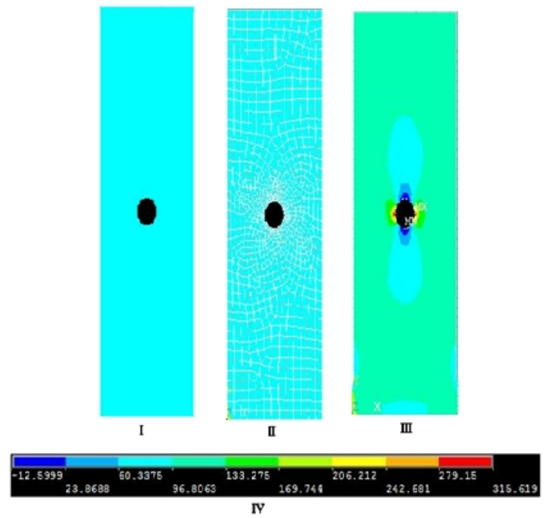


Fig. 1. A plate with d/w ratio 0.2 (I) Geometry,(II) Meshing ,(III) Nodal Stress Distribution ,(IV) Stress Value in the plate.

Case II Effect of Size increase on the stress concentration around the hole in the plate in this case we will consider the d/w ratio equal to 0.3 means the diameter of the hole is 30 mm. The FEA analysis is carried out for finding the maximum stress and that stress concentration factor. Nominal stress is calculated after these.

Hence the results show that the maximum stress is obtained at the edge of the hole. The maximum value of the stress is 335.8 N/mm^2, nominal stress is equal to 142.8 N/mm^2 and the stress concentration at the hole is 2.35 so by the increase in the size of hole the maximum value of the stress is increased and stress concentration factor is decreased around the hole in the plate.

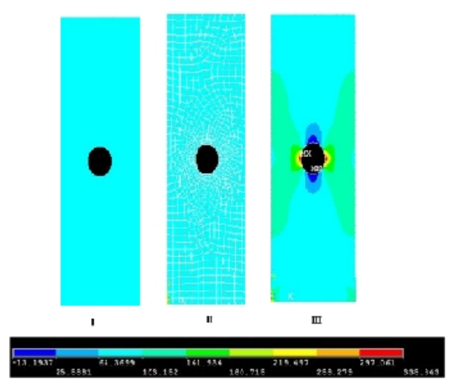


Fig. 2. A plate with d/w ratio 0.3 (I) Geometry, (II) Meshing, (III) Nodal Stress Distribution, (IV) Stress Value in the plate.

Case III Now the d/w is 0.35 means the diameter of hole is 35 mm at the centre of the plate the stress is applied to the plate and stress distribution and stress concentration factor are studied and it is found that the maximum value of stress is equal to 352.16 N/mm^2 and nominal stress equal to 153.8 N/mm^2 and stress concentration factor is equal to 2.28 and the value of SCF is again decrease.

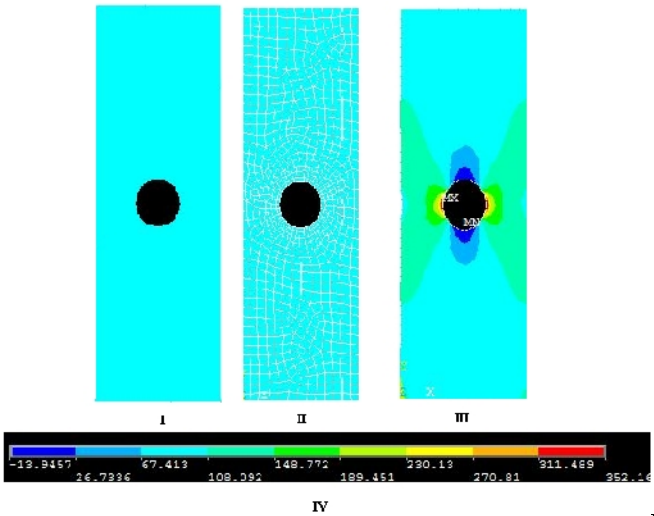


Fig. 3. A plate with d/w ratio 0.35 (I) Geometry,(II) Meshing ,(III) Nodal Stress Distribution ,(IV) Stress Value in the plate.

Case IV In this case the value of diameter of hole in the plate is 40 mm and d/w ratio is become 0.4

Stress distribution based on FEM analysis is given above and maximum stress value and nominal stress is found from the solution. Stress Concentration factor is found from ANSYS results as shown above. The value of Maximum stress is 378.18 N/mm^2 and value of nominal Stress is 166.66 N/mm^2 so Stress concentration factor is becomes 2.26.

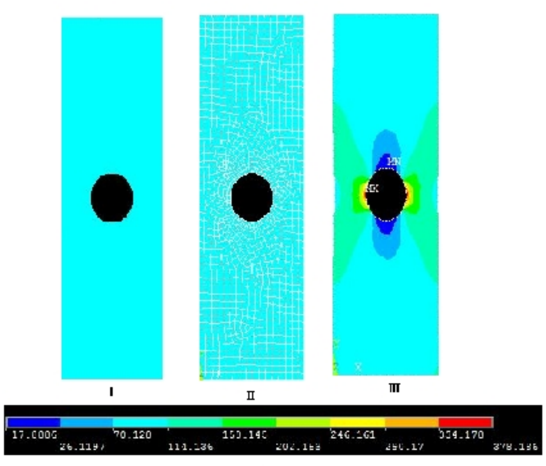


Fig. 4. A plate with d/w ratio 0.40 (I) Geometry,(II) Meshing ,(III) Nodal Stress Distribution ,(IV) Stress Value in the plate.

Case V The d/w ratio of the plate is 0.45 so diameter of hole is now 45 mm and FEM analysis of plate is Carried out. The maximum Stress is equal to 399.3 N/mm^2 and nominal stress is equal to 181.81 N/mm^2 so the value of Stress Concentration Factor (SCF) became 2.19 and as we see the value of SCF is decrease as the size of hole is increase in the centre of plate.

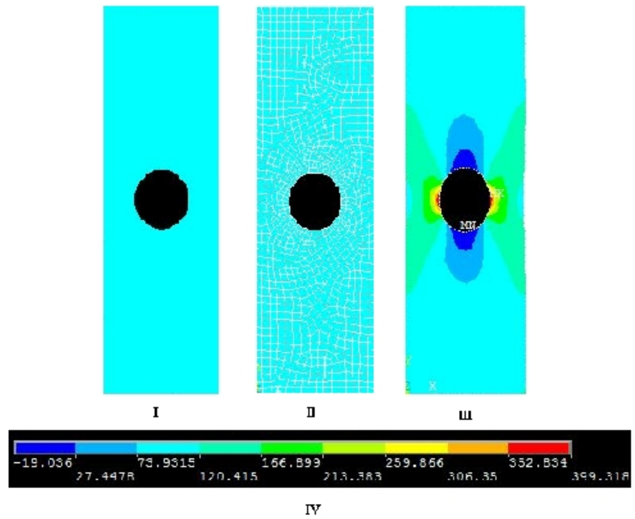


Fig. 5. A plate with d/w ratio 0.45(I) Geometry,(II) Meshing ,(III) Nodal Stress Distribution ,(IV) Stress Value in the plate .

Case VI The d/w ratio of the plate is 0.50 in this case so the diameter of hole in the plate become equal to 50 mm.

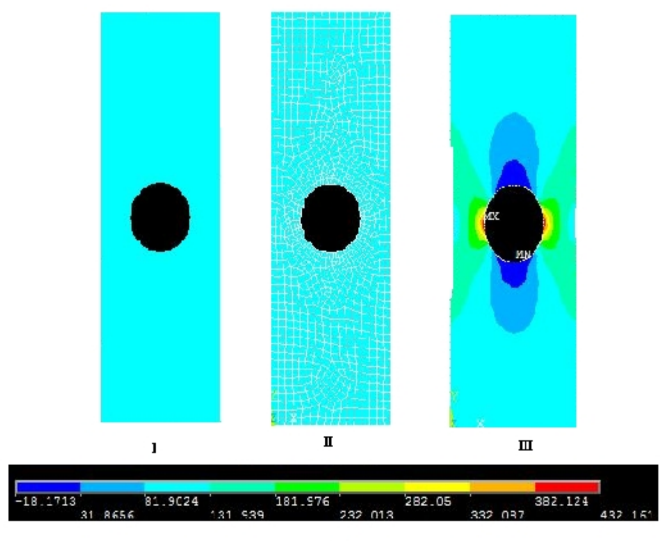


Fig. 6. A plate with d/w ratio .50 (I) Geometry,(II) Meshing ,(III) Nodal Stress Distribution ,(IV) Stress Value in the plate .

FEA analysis is carried out using software. The value of Maximum stress is found 432 N/mm^2 and the Value of Nominal stress is 200 N/mm^2 so the stress concentration factor in the plate near discontinuity is equal to 2.16.

In this case the maximum value of stress is found 473.25 N/mm^2 and Nominal value of stress is 222.2 N/mm^2 so the stress concentration factor is equal to 2.13.

The Stress Concentration Factor at different d/w ratio are plotted on the graph. The graph shows different values of d/w ratio to values of SCF respectively as shown above.

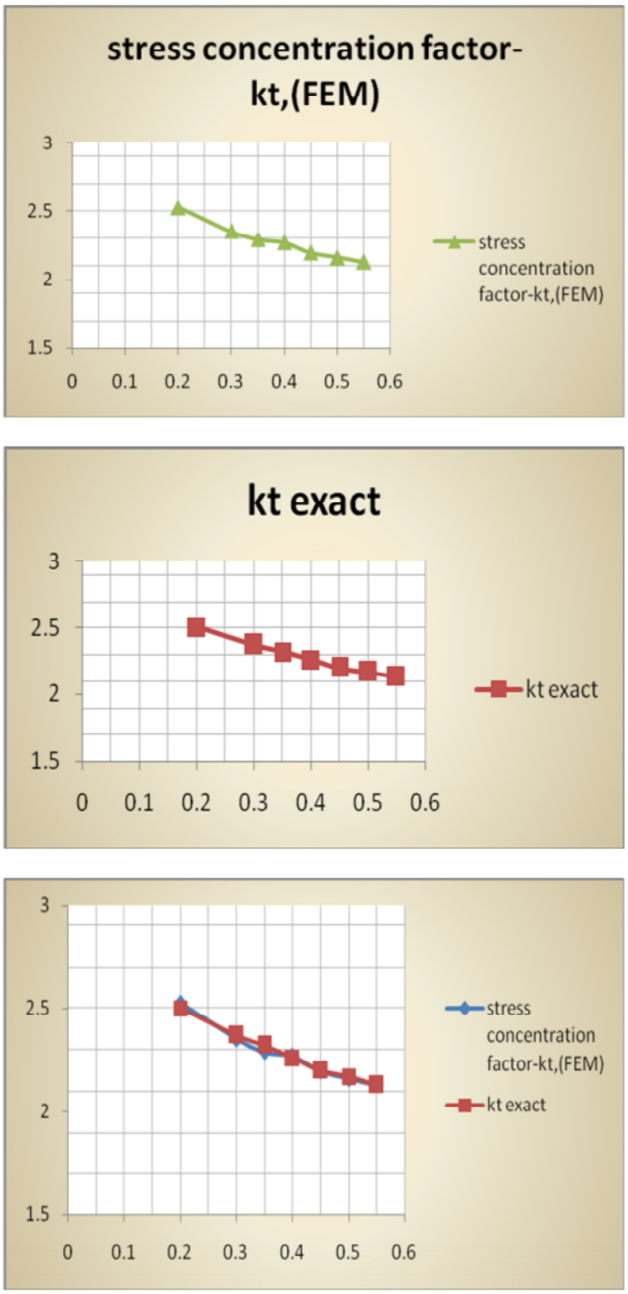


Fig. 7. Graphical representation of the Variation of SCF v/s D/W ratio of the plate and caparison with exact data.

The results are compared with Peterson charts for same d/w ration given in Shigly's Mechanical Engineering Data book (Published by McGraw Hill).

IV. EXTENDED FINITE ELEMENT METHOD (XFEM) SOLUTION

Extended Finite Element method is applied for the same Numerical problem as we seen in above example. For FEA analysis there is essential requirement of conformal mesh and the discontinuity is counted during mesh generation or we can say that discontinuity is included during mesh generation. Mesh is generated during every step of the analysis this is a very tedious work and more time consuming. In Extended Finite Element Method there is no requirement of the conformal mesh. Discontinuity will be not included during the mesh generation. Discontinuity will be handled by enrichment function and located by Level set function. So there is not any requirement of including the discontinuity during mesh generation.

Case I The solution by Extended Finite Element Method for d/w 0.2 for the maximum value of stress and nominal value of stress is calculated. Then Stress Concentration Factor is calculated and compare with the solution of FEA analysis obtained from ANSYS. XFEM Solution is obtained with the help of MATLAB. For above problem the maximum value of stress given by XFEM solution is equal to 300 N/mm^2 and value of nominal stress equal to 125 N/mm^2. The stress concentration factor given from above solution by XFEM is equal to 2.4 and maximum stress occurs at the edges of hole in the plate.

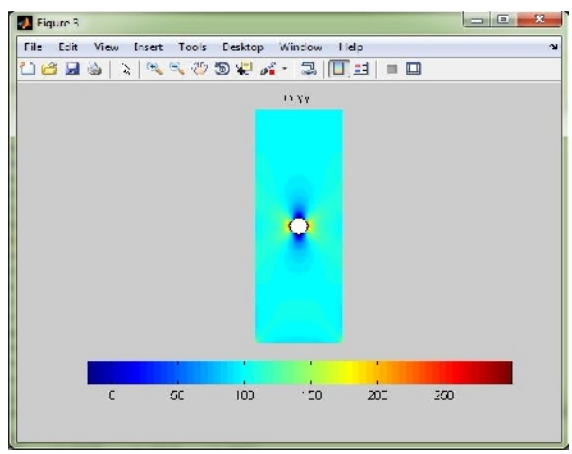


Fig. 8. XFEM solution for the Stress Distribution in the plate having d/w value to 0.2.

Case II When the d/w ratio is equal to 0.3 stress distribution and value of stress concentration factor is found in the plate with the help of XFEM.

By XFEM the maximum stress is found at the edges of the hole and value is found 324.25 N/mm^2 and Value of Nominal stress is found 142.8 N/mm^2, the value of SCF is equal to 2.34 **Case III** In this case the d/w ratio of the dia. of hole in the plate and width of the plate is 0.35 means that the diameter of hole is 35 mm. The stress is applied to the plate.

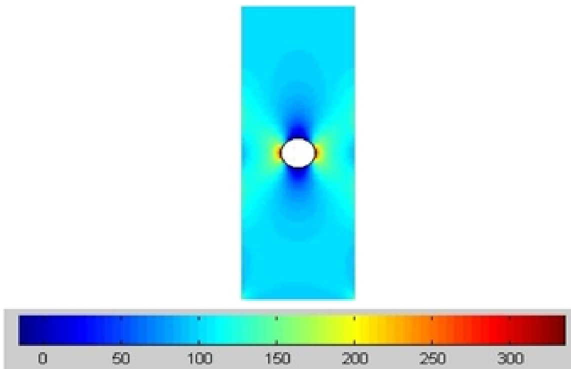


Fig. 9. XFEM solution for the Stress Distribution in the plate having d/w value to 0.3.

From above solution the maximum value of stress is found at the edge of hole and the value of maximum stress is equal to 352.53 N/mm^2 and Nominal stress is equal to 153.8 N/mm^2 hence the SCF is 2.29 in this case.

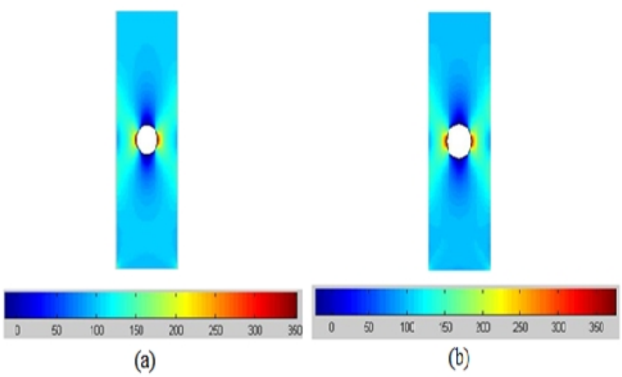


Fig. 10. (a) Stress distributions in the plate having d/w ratio equal to 0.35 by XFEM, (b) Stress distributions in the plate having d/w ratio equal to 0.40 by XFEM.

Case IV The d/w ratio in this case is equal to 0.40 so the diameter of hole in the plate is equal to 40 mm and XFEM is applied to find out the stress distribution in the plate and stress concentration factor is also calculated. The stress distribution in the plate shown in Fig. 10(b) and maximum stress occoured at the edges of the hole in the stress applied direction hence by solution of XFEM method the maximum value of stress is equal to 374.45 N/mm^2 and the nominal value of stress is 166.66 N/mm^2. The stress concentration factor is 2.25 when d/w ratio is equal to 0.40.

CaseV In this case the Diameter of the hole in the plate is 45 mm so the D/W ratio is equal to 0.45. Now the effect of Stress distribution is carried out with the help of XFEM method over the plate and as we know that the maximum value of stress is occuring at the edges of the holes in the plate The maximum value of the stress in this case is 401.15 N/mm^2 and the nominal value of the stress is 181.81 N/mm^2, so the value of stress concentration factor is 2.20 in this case which is near to the hole in the plate.

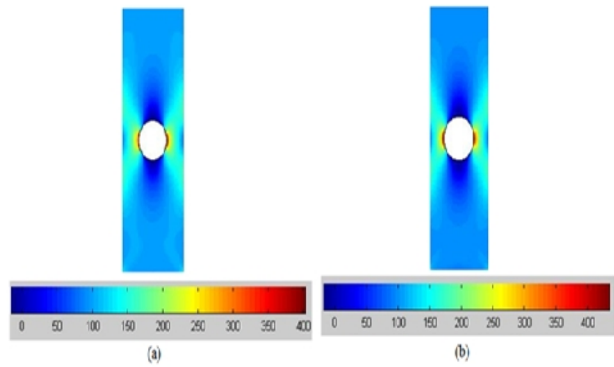


Fig. 11. (a) Stress distributions in the plate having d/w ratio equal to 0.45 by XFEM, (b) Stress distributions in the plate having d/w ratio equal to 0.50 by XFEM.

CaseVI In this case the d/w ratio i.e, the diameter of hole to the width of plate is 0.50 and diameter of hole in this case is equal to the 50 mm and again by appling XFEM method the stress distribution is calculated . The distribution of stress over the plate is as shown in Fig. 12(b) as shown in Fig. the maximum value of stress is at the edge of hole and the maximum value of stress is equal to the 435.12 N/mm^2 and the nominal stress in this case is equal to the 200 N/mm^2 so the value of stress concentration factor is 2.17 in this case.

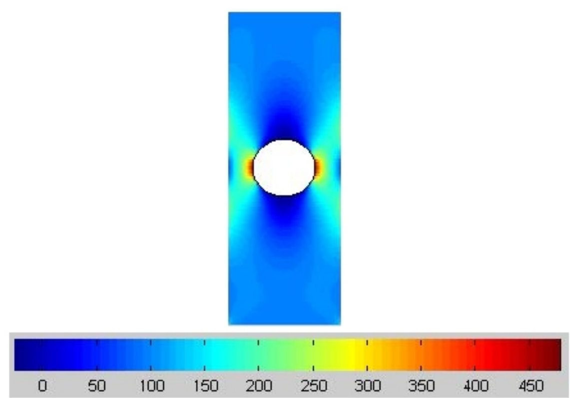


Fig. 12. Stress distributions in the plate having d/w ratio equal to 0.55 by XFEM**.**

V. REDUCTION STRESS CONCENTRATION FACTOR (SCF)

The way to reduce the Stress Concentration Factor (SCF) around a central circular hole in a uniaxial loaded plate is, "Introducing smaller auxiliary holes above and below the original hole. These Auxiliary holes above and below the original hole help to smooth the stress path to help the original hole."

Hence we will take three cases which are used to explain the reduction of Stress concentration Factor in the plate.

Case I In this case we will take the d/w ratio equal to 0.3 means that the diameter of hole is equal to 30 mm and for reduction of Stress Concentration factor we will introduce the auxiliary hole above and below the central hole of the plate for providing the smooth path for the stress distribution in the plate.

The diameter of the auxiliary hole we will take equal to 20 mm and the distance between the centre hole and the auxiliary hole we will take 40 mm so the stress distribution in the plate will be solve by XFEM method.

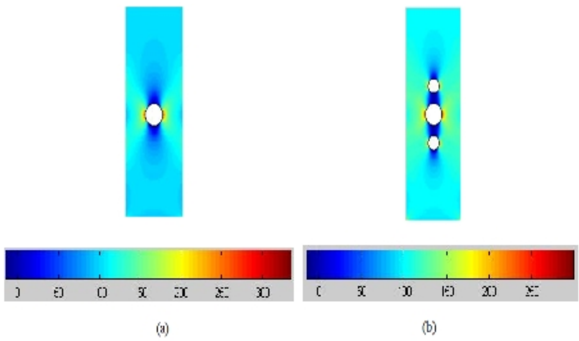


Fig. 13. (a) Stress Distribution in the plate having d/w0 .30, (b) Stress Distribution in the plate having multiple holes with d/w0.30.

From the solution we obtained the maximum value of stress is equal to 298.99 N/mm^2 and the nominal value of stress is equal to 142.8 N/mm^2 so the value of stress Concentration factor (SCF) is in this multiple hole case is equal to 2.09 but for the same d/w ratio i.e. in the previous case the plate having only central hole having diameter equal to 30 mm as shown in Fig. 13. (a) the value of SCF is 2.35 so the percentage (%) decrease in SCF is 11.06% hence we can say that by providing the auxiliary holes above and below the central hole in the plate we can reduce the stress concentration factor.

Case II In this case we will take the d/w ratio equal to 0.40 means the diameter of hole in the plate is equal to 40 mm and we will add the two auxiliary holes one above and one below to the central hole for reducing the SCF in the plate around the hole.

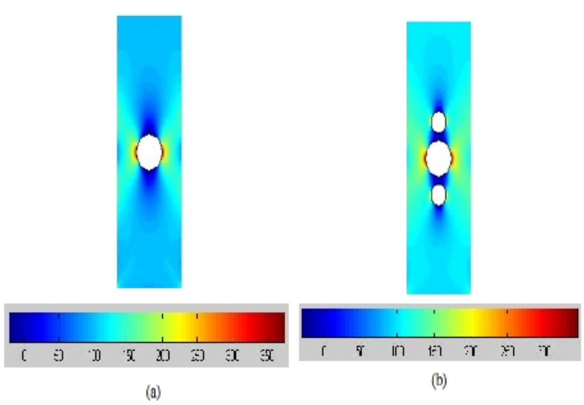


Fig. 14. (a) Stress Distribution in the plate having d/w ratio 0.40, (b) Stress Distribution in the plate having multiple hole with d/w 0.40.

The diameter of both the auxiliary holes is equal to 24 mm and the spacing b/w the centre of auxiliary hole and the centre of central hole is 40 mm. These auxiliary holes provide the smooth path for stress distribution in plate. Hence these will help to reduce the Stress Concentration Factor and the value of SCF in case of only central hole is equal to 2.26 but as we provide the auxiliary holes the maximum stress become 345.56 N/mm^2 and the value of nominal stress is 166.66 N/mm^2 so stress concentration factor is equal to 2.07. The % reduction in SCF is equal to 8.04%.

Case III The diameter of the central hole in this case is taken equal to 50 mm so the ratio of d/w is 0.50 in this case and the diameter of auxiliary hole in this case is taken equal to 44 mm for reducing the value of Stress Concentration Factor. The spacing distance between centre of main hole and auxiliary hole is taken 55 mm in this case.

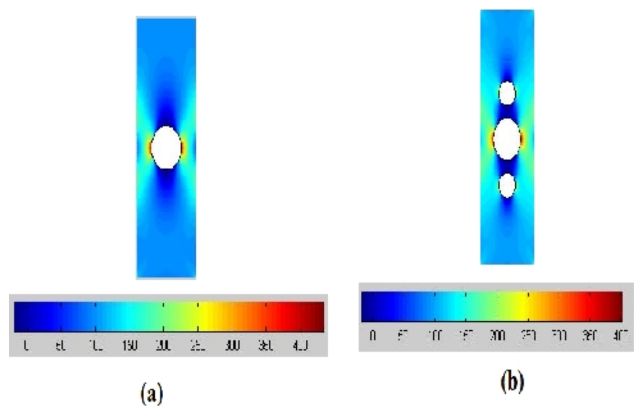


Fig. 15. (a) Stress Distribution in the plate having D/W ratio 0.50, (b) Stress Distribution in the plate having multiple hole with d/w 0.50.

In the presence of only central hole the value of Stress Concentration Factor is 2.18 but after providing the auxiliary holes the value of maximum stress become 401.36 N/mm^2 and the value of nominal stress found 200 N/mm^2 so the value of SCF become 2.01 and the percentage reduction in the value of SCF is 7.79% in this case.

The whole above observation can also be explained with the help of graphical representation as given below. The graph is plotted b/w the value of SCF and the d/w ratio of the plate for both the case as above i.e. for single hole in the plate as well as the multiple holes (auxiliary holes) into the plate. From all above calculation we can say that the value of stress concentration factor is decreasing by applying the auxiliary holes above and below the central or main hole in the plate.

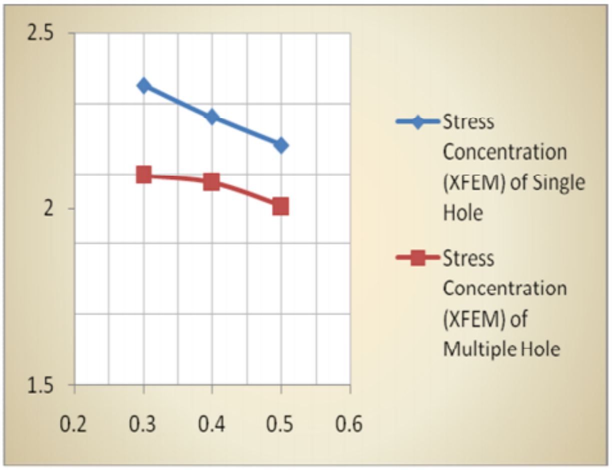


Fig. 16. Graphical representation of the reduction in SCF.

VI. EFFECT OF SPACING DISTANCE BETWEEN MAIN HOLE AND AUXILIARY HOLES

Now considering the spacing distance (a) between the main hole and the auxiliary holes and take the ratio of spacing distance and the width of the plate like 0.30 and 0.35 etc. and we will study the variation between the a/w ratio and the value of Stress Concentration Factor by keeping the diameter of auxiliary hole fixed at a constant value equal to 20 mm the variation between d/w ratio also equal to take a/w ratio.

Case I when a/w is equal to 0.30 that means the spacing distance between the main and auxiliary holes is 30 mm and the diameter of main hole is 30 mm and diameter of auxiliary hole is equal to 20 mm. In this case the maximum value of stress in the plate and also the value of nominal stress are calculated.

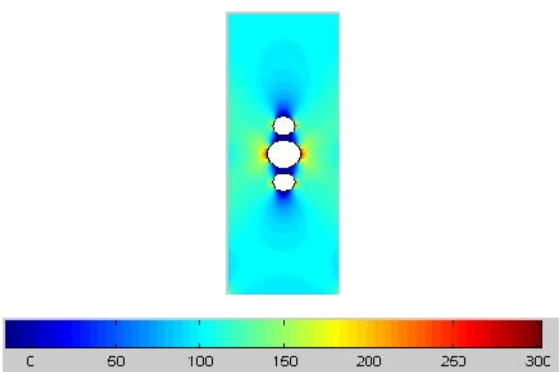


Fig. 17. The stress Distribution in the plate having a/w and d/w ratio 0.30.

The maximum value of stress in this plate is equal to 278.55 N/mm^2 and the nominal stress is 142.8 N/mm^2 so the value of stress concentration factor is 1.95.

Case II The ratio of a/w in this case is equal to 0.35 and also the ratio of d/w is equal to 0.35 so the stress analysis is carried out for this case by XFEM method and the maximum stress will be calculated.

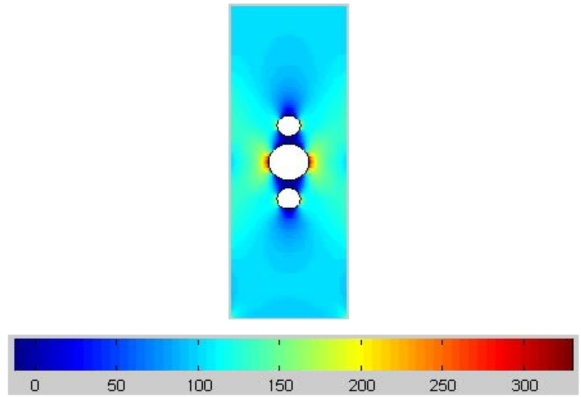


Fig. 18. The stress Distribution in the plate having a/w and d/w ratio 0.35.

In this case the spacing distance between the main and auxiliary hole is 35 mm and diameter of auxiliary hole is again 20 mm and the diameter of main hole is 35 mm. The maximum value of stress is equal to 311.62 N/mm^2 in the plate and the value of nominal stress is equal to 153.8 N/mm^2. The value of stress concentration factor is equal to 2.02.

Case III In this case the ratio of spacing distance between main and auxiliary holes to width of plate is 0.40 so the distance is 40 mm and d/w ratio is also 0.40 so the diameter of main hole is 40 mm and diameter of auxiliary hole is kept constant at 20 mm. The stress distribution in the plate is as shown below.

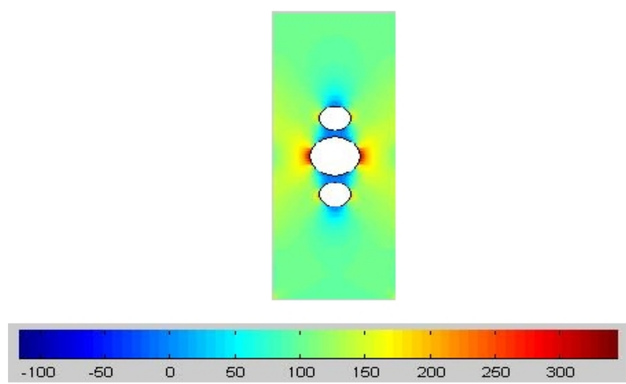


Fig. 19. The stress Distribution in the plate having a/w and d/w ratio 0.40.

From solution the maximum stress is 342.41 N/mm^2 and the nominal stress is equal to 166.66 N/mm^2 hence the SCF for this case is equal to 2.05.

Case IV The a/w and d/w ratio in this case is taken equal to 0.45 and the diameter of auxiliary hole is 20 mm and the diameter of main hole is 40 mm and the spacing distance is also equal to 45 mm. The stress in the plate is given by XFEM as shown below

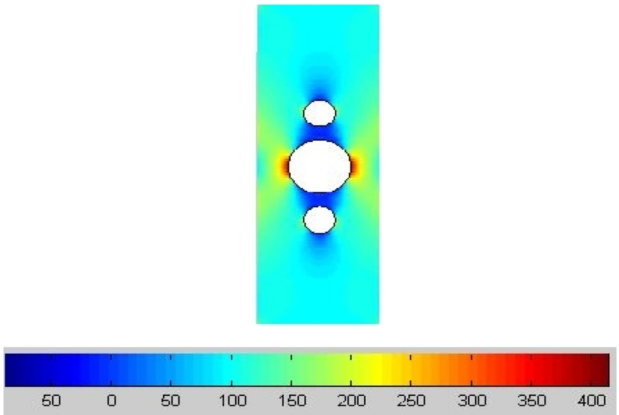
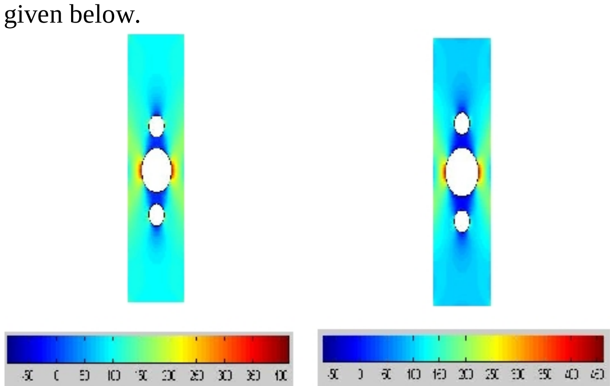


Fig. 20. The stress Distribution in the plate having a/w and d/w ratio 0.45.

Case V In this case the ratio of spacing distance between main and auxiliary holes to plate width is taken 0.50. The spacing distance is 50 mm and diameter of main hole and auxiliary hole is 50 mm and 20 mm respectively and the stress distribution in the plate is given below.



(a) (b)

Fig. 21. (a) The stress Distribution in the plate having a/w and d/w ratio 0.50, (b) The stress Distribution in the plate having a/w and d/w ratio 0.55.

The maximum value of Stress is 414.33 N/mm^2 and the nominal stress is equal to 200 N/mm^2 so the Stress Concentration Factor is 2.07.

Case VI The a/w and d/w ratio in this case is equal to 0.55 so we can say that the spacing between the main hole and the auxiliary holes is 55 mm. The diameter of main hole is 55 mm and the diameter of auxiliary hole is 20 mm. The stress Distribution in the plate is shown in Fig. 21 (b). The Maximum value of Stress is 461.19 N/mm^2 and the Nominal Stress is equal to 222.22 N/mm^2 so in this case the SCF is 2.07.

The Graphical Representation of above all case is as shown below.

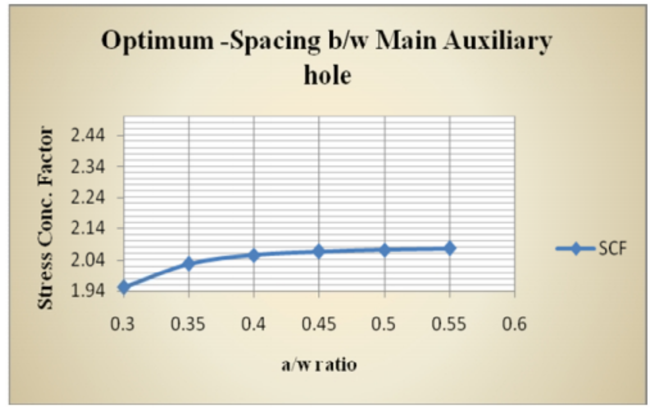


Fig. 22. Graph b/w SCF and a/w.

VI. CONCLUSION AND FUTURE WORK

Finite Element Method is a numerical approach which is used in the engineering for analysis. In this work the analysis of plate with discontinuity like hole in it is analyzed by FEM and it is noticed that a conformal mesh is required at every stage in FEM. Removing this difficulty XFEM is imposed on it. Extended Finite Element Method (XFEM) is a Partition of Unity (POU) Method and XFEM is enhancement of the approximation of space with enrichment function. In this thesis work the XFEM is used to solve the 2D problem having discontinuity like hole in the domain. The effect of hole in the plate with varying size has been studied with respect to plate width and analyzed in detail.

From the present work the following conclusions are drawn

- It is noticed that the conformal mesh will be required at every stage in FEM but we can overcome this difficulty in XFEM.
- The value of Stress Concentration Factor (SCF) for the plate with hole of various d/w ratios has been computed by XFEM.
- The Number of elements in the FEM analysis is 10075 for the plate having d/w ratio equal to 0.50 but in XFEM for the same plate the required number of elements is only 4800 for same results.
- The percentage reduction in the SCF is 7.79 for d/w ratio of 0.50 by providing the auxiliary holes of radius 22 mm on both sides of main hole.

XFEM can be used to study the effect of hole on the bi material interface problem.

XFEM can be used to analyze the cracks generated through the hole in structure.

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